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Hall-MHD surface waves in flowing solar-wind plasma slab

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We study the influence of a steady flow velocity on the dispersion characteristics of Hall-magnetohydrodynamic forward and backward sausage and kink surface waves travelling along a solar-wind plasma slab. Both surface modes, being Alfvén waves in an immovable layer, become super Alfvénic ones in flowing slab with increased dispersion at small wave numbers for forward waves and at large wave numbers for backward waves, respectively.

1. Motivation

Recent observations made by two *HELIOS* spacecrafts have confirmed the existence of fine structures in high-speed solar-wind flows. Satellite measurements of plasma characteristics such as the magnetic field, the velocity and density of the plasma are important to understand the different plasma wave modes which may occur. However, wave analysis requires further information and special tools in order to be able to identify which set of modes is contributing to observed wave features [1]. While the magnetohydrodynamic (MHD) waves (acoustic, Alfvén and magnetosonic modes) possess frequencies much below the ion cyclotron frequency ω_{ci}, the frequency range of Hall-MHD waves is expanded up to ω_{ci}. Recall that Hall-MHD theory is relevant to plasma dynamics occurring on length scales shorter than an ion inertial length ($l_{\rm Hall} < c/\omega_{pi}$, where c is the speed of light and ω_{pi}—the ion plasma frequency) [2]. Assuming the solar-wind plasma is structured due to different plasma/mass densities inside and outside the slab [3], the Hall length scale for a solar-wind flux tube at 1 AU with ambient magnetic field $B_0 = 3.6 \times 10^{-9}$ T (and accordingly ion cyclotron frequency $\omega_{ci}/2\pi = 0.055$ Hz), electron number density $n_e = 2 \times 10^6$ m^{-3}, electron temperature $T_e = 2 \times 10^5$ K is $l_{\rm Hall} = 160$ km. The Alfvén speed is $v_{\rm A} = 56$ km s^{-1}, sound speed being $v_s = 52$ km s^{-1} and hence the plasma β equals approximately unity. The slab steady flow speed is $v_0 \approx 400$ km s^{-1}.

2. Basic equations and relations

Consider a flowing plasma slab of uniform density $\rho_{\rm o}$ and thickness $2x_0$, bounded by immovable plasmas of densities $\rho_{\rm e}$, the interfaces being the surfaces $x = \pm x_0$. The uniform magnetic field $\mathbf{B}_0$ and the steady flow velocity $\mathbf{v}_0$ point in the z direction. The wave vector $\mathbf{k}$ lies also along the z axis and its direction is the same as that of $\mathbf{B}_0$ and $\mathbf{v}_0$ for forward waves and opposite for backward waves, respectively. The basic equations for Hall-MHD waves are the linearized equations governing the evolution of perturbed plasma density $\delta\rho$, pressure δp, fluid velocity $\delta\mathbf{v}$ and wave magnetic field $\delta\mathbf{B}$ [4],

$$\frac{\partial}{\partial t}\delta\rho + (\mathbf{v}_0\cdot\nabla)\,\delta\rho + \rho_0\nabla\cdot\delta\mathbf{v} = 0,$$

$$\rho_0\frac{\partial}{\partial t}\delta\mathbf{v} + \rho_0\,(\mathbf{v}_0\cdot\nabla)\,\delta\mathbf{v} + \nabla(\delta p + \frac{1}{\mu_0}\mathbf{B}_0\cdot\delta\mathbf{B}) - \frac{1}{\mu_0}\,(\mathbf{B}_0\cdot\nabla)\,\delta\mathbf{B} = 0,$$

$$\nabla\cdot\delta\mathbf{B} = 0,$$

$$\frac{\partial}{\partial t}\delta\mathbf{B} - (\mathbf{B}_0\cdot\nabla)\,\delta\mathbf{v} + \mathbf{B}_0\nabla\cdot\delta\mathbf{v} + (\mathbf{v}_0\cdot\nabla)\,\delta\mathbf{B} + l_{\rm Hall}\frac{v_{\rm A}}{B_0}\mathbf{B}_0\cdot\nabla\nabla\times\delta\mathbf{B} = 0,$$

$$\frac{\partial}{\partial t}\delta p + (\mathbf{v}_0\cdot\nabla)\,\delta p + \gamma p_0\nabla\cdot\delta\mathbf{v} = 0,$$

where $v_{\rm A} = B_0/\left(\mu_0\rho_0\right)^{1/2}$ and $\gamma = 5/3$. The pressure perturbation δp is related to the mass density perturbation $\delta\rho$ via $\delta p = v_s^2\delta\rho$, where $v_s = \left(\gamma p_0/\rho_0\right)^{1/2}$ is the sound speed. Following the way of solving the above set of equations developed in Ref. [4], after Fourier transforming all perturbed quantities $\propto g(x)\exp(-i\omega t + ikz)$, we derive two coupled second order differential equations for δv_x and δv_y. All other perturbed quantities are expressed in terms of δv_x and δv_y. By applying the boundary conditions for continuity of $\delta v_x/\left(\omega - \mathbf{k}\cdot\mathbf{v}_0\right)$, the full perturbed pressure $\delta p_{\rm total}$ (kinetic + magnetic), the y-component of perturbed wave electric field δE_y, and the x-component of perturbed electric displacement $\delta D_x = \varepsilon_0\left(K_{xx}\delta E_x + K_{xy}\delta E_y\right)$ (where K_{xx} and K_{xy} are the low-frequency components of the plasma dielectric tensor [5]) at the interfaces, we arrive at the dispersion relations of Hall-MHD sausage and kink surface waves presented symbolically in the form

$$\mathcal{D}\left(\omega, k, v_0, B_0, l_{\rm Hall}, x_0, \beta, \rho_{\rm o}, \rho_{\rm e}\right) = 0.$$

Solving numerically the waves' dispersion relations we get the wave phase velocity ω/k (being Doppler-shifted inside the slab), normalized with respect to the Alfvén speed $v_{\rm Ao}$, as a function of the dimensionless wave number kx_0 with four entry parameters, notably the ratio $\rho_{\rm o}/\rho_{\rm e} = 4$, the plasma $\beta = 1$, the ratio $l_{\rm Hall}/x_0 = 0.4$, and the Alfvén Mach number $v_0/v_{\rm Ao}$ lying in the range 7–8 (because the steady flow velocity v_0 is not a constant, but swinging around the aforementioned value of 400 km s^{-1}).

3. Results and discussion

It is well known that sausage and kink surface waves are normal modes of a spatially bounded MHD flux tube. For a flowing plasma slab one can distinguish two kind of waves: forward and backward ones assuming that the positive direction is defined by the steady flow velocity $\mathbf{v}_0$. Figures 1 and 2 show the dispersion curves of forward sausage and kink waves.

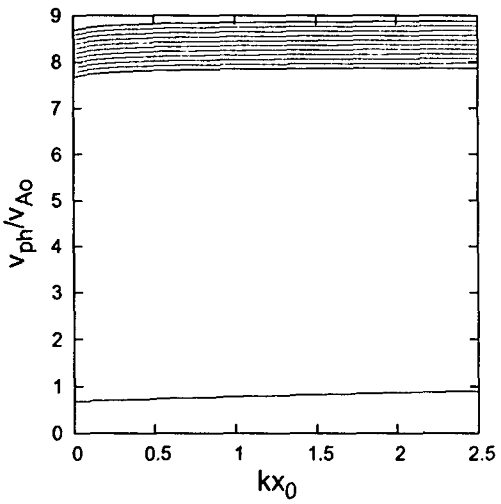

Figure 1: Forward Hall-MHD sausage waves

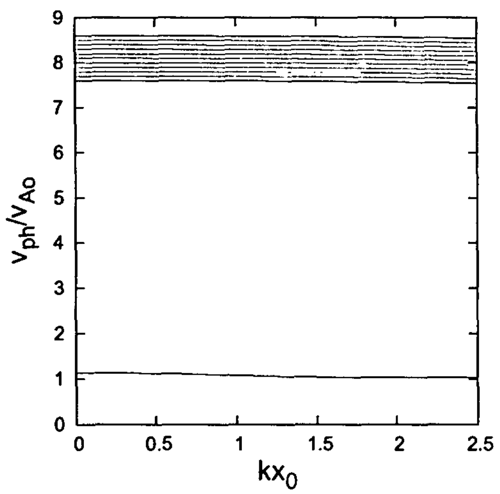

Figure 2: Forward Hall-MHD kink waves

Next two figures 3 and 4 present the dispersion characteristics of backward waves. It is seen, first, that all waves, being Alvfén waves without flow, become super Alfvénic ones with flow. Second, the dispersion of forward waves is slightly modified with small kx_0's by the flow speed while that of the backward waves is visibly changed for large dimensionless wave numbers. Moreover, the phase velocity of each surface wave can be presented in the form [6]

$$\frac{\omega}{k} = \frac{\rho_o}{\rho_o + \rho_e} v_0 \pm \text{Hall-MHD } v_{\rm ph},$$

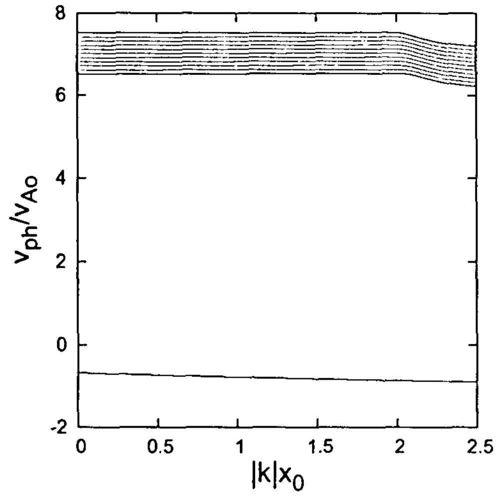

Figure 3: Backward Hall-MHD sausage waves

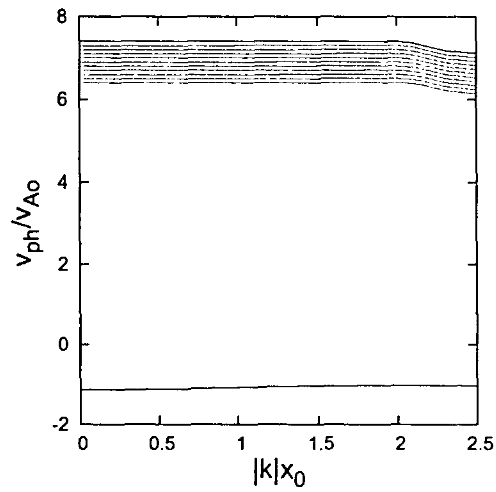

Figure 4: Backward Hall-MHD kink waves

where Hall-MHD $v_{\rm ph}$ is the phase velocity which the wave possesses in an immovable plasma slab.

4. References

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